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Abstract: We propose an ethical viewpoint based on the possibility of the realization of the worst-case scenario in order to reduce future generations risks in terms of discounting. Applied to the question of conservation of a renewable resource, we show that an economy, where the social planner takes into account the possibility that at an uncertain date the discount rate could change to its minimum possible value, could lead to a better conservation of the resource and modify the position of the sacrificed generations. Finally, our model suggests to apply the lowest possible discount rate immediately for long term environmental projects.

Résumé: Nous proposons une approche basée sur la possibilité de la réalisation du pire scénario pour réduire les risques encourus par les générations futures en termes de taux d'actualisation dès maintenant. Appliquée à la question de la préservation d'une ressource renouvelable, nous montrons qu'une économie dans laquelle le planificateur social prend en compte la possibilité d'un changement du taux d'actualisation à sa valeur minimale à une date future incertaine peut conduire à une meilleure préservation de la ressource et modifier la position des générations sacrifiées. Finalement, notre modèle suggère d'appliquer le taux le plus faible possible dès aujourd'hui pour des projets environnementaux de long terme.

Keywords: Discounting; Environment; Uncertainty; Preservation of natural resources; Intergenerational equity.

Mots clés: Escompte; Environnement; Incertitude; Préservation; Equité intergénérationnelle.

Classification JEL: O4, Q2, D80

1 Introduction

Long-lasting environmental phenomena such as global climate change, radioactive waste disposal, minerals depletion, and loss of biodiversity have raised the question of our ability to discount properly environmental projects which effects will be spread out over hundred of years. This question involve to a large extent the uncertainty conveyed by the distant future as it may have an impact on both the future of society as featured by Gollier (2002) and Weitzman (1999) and the attitude of future generations towards environmental assets.

In the literature, the effect of uncertainty on future preferences on conservation decisions has been mainly investigated from the viewpoint of changes on the utility function. The main question addressed is to consider the possibility that future generations have stronger preference for environmental goods. That is to say, future generations may value natural resources and environmental assets quite differently than we do today. In seminal studies of this issue, Beltratti, Chichilnisky and Heal (1993), and Ayong Le Kama (2001) study the effect of change in preferences in the future at an unknown date on the optimal use of natural resource. They consider a model in which the unique source of welfare is consumption, derived from the depletion of an exhaustible natural resource. They show that when the central planner expects that the preferences of future generations will be, on average, more in favor of the environment than we do, the uncertainty on future preferences leads to a more conservative attitude in terms of the use of resource stocks[1].

The appropriate discounting of the very long term also questions strong beliefs about what constitutes ethical behavior of current generations toward the future of generations to come. This is about the willingness of members of current generations to bear the cost of actions that reduce risks faced by future generations on the debate on discounting the distant future and the intergenerational resource allocation. This led Arrow (1999) to assert that the problem of discounting for projects with payoff in the far future is largely ethical. This paper is also made from an ethical viewpoint and argues that it is important to clarify the normative grounds under which the choice of social discount rate is made. We propose an approach based on the possibility of the realization of the worst-case scenario in order to reduce future generations risks in terms of discounting.

Planning the distant future requires taking into account accurate assumptions on the evolution of

[1] Beltratti, Chichilnisky and Heal (1998) develop also the case in which the stock of exhaustible resource and consumption enter in a separable way into the utility function. They reach the same result since in that case, the marginal utility of consumption does not depend on the preference for the resource and hence it is not affected by the changes in preferences. Besides, Ayong Le Kama and Schubert (2004) develop the non separable case and show that two kinds of behavior are possible for the decision maker. If the economy has poor prospects of growth and a bad environmental quality, it will rather adopt a precautionary behavior and be more conservative. However, if on the contrary, the growth path without change in preferences is favorable and the environmental quality fair, the society will rather adopt a behavior of insurance against later depravations, consisting in consuming a great deal now at the expenses of the environmental quality.

relevant variables such as growth rates, technologies, resource available in order to value objectives. Regarding environmental and economic variables, one may consider, on the one hand, an optimist evolution with high growth rates, clean technologies, and new purpose technologies, etc.... On the other, one may consider a pessimist evolution with low growth rates, side effects of natural resource scarcity, deterioration of the environment and decreasing returns of scale, etc.... The future evolution of these variables is very important to well define the appropriate social discount rate. It can be difficult to define a probability distribution on the future values of these variables for such a long of time period. Instead, it may be more intuitive to adopt the worst-case attitude to define the social discount rate. That means to consider that the evolution of the variables is systematically adverse and therefore to select the smallest one. The rationale behind this approach is to say that since the worst case scenario is possible, and that if it turns to happen future generations will support the highest possible damage. Therefore, it is then fair to apply to the social welfare function the discount rate that allocates the highest weight to future generations well-being[2]. This argument is also consistent with regard to the precautionary principle and intergenerational equity[3].

Our approach leads to another rationale for discounting the far distant future at its lowest possible rate and therefore complements the analysis of Weitzman (1998) and Gollier (2002).

Applied to the question of the optimal use of renewable resource, we show in this paper that when the social planner takes into account the possibility that the discount rate could change to its minimum possible value at an uncertain date, an economy would experience a better conservation of the resource. Moreover, this raises the question of the timing and the impact on the economy of the change in the social discount rate that involves which generations bear the sacrifice. In one possible case, it will not be the current generations, nor the later generations, but those that comes just after the change in discount rate occurred that will make the sacrifice for the transition. Moreover, the time slot for which this generation would be sacrificed could be reduced if the growth rate of the economy adopting the worst-case scenario attitude is significantly higher than the one in the determinist case. In another possible case, the economy which takes into account the possibility of the change in the discount factor at an uncertain date will be more conservative in terms of resource consumed till the change occurs. The first generations are then heavily sacrificed to pay attention to the interest of future ones.

[2] The worst-case scenario we developed is based on the *minimum principle* approach and can be declined in an equivalent *maximin* version. Therefore, one can give ethical interpretations to decisions made from the worst-case scenario approach in a *Rawlsian* ethic framework of the choice of principles in uncertainty context, the *veil of ignorance*.

[3] One can interpret our approach in terms of precautionary principle in the sense that we should not let future generations bear the risk in terms of discounting since uncertainties belonging to the future are not resolved.

Implicitly, our approach suggests to make the change in discount rate the sooner the better and therefore to apply the lowest possible discount rate today. This result can be confronted to the recommendation of the French Commissariat Général au Plan which proposes to reduce the public discount rate from 8% to 4% at the horizon of 30 years, 3% for 100 years and about 2% beyond that.

2 The determinist case

We consider an economy in which the stock of the resource, S, is depleted by consumption c, but regenerates itself at the constant rate $M > 0$. Thus, its dynamic is written as:

$$\dot{S}_t = MS_t - c_t. \tag{1}$$

At time t, society derives utility from consumption and the amenity of the stock of the resource according to the felicity function $u(c_t, S_t)$ which is assumed to be increasing, strictly concave with respect to its arguments and twice continuously differentiable. Thus we assume a utility function of the Cobb-Douglas form, such as: $u(c, S) = \ln c + \phi \ln S$, where ϕ stands for the relative preference for the resource stock.

In the determinist case, the social planner applies a constant, positive and certain discount rate all together of the infinite horizon of the economy. Therefore, the problem of the social planner can be formulated as:

$$\mathcal{P}(1) \quad \max_{c,S} \int_0^\infty e^{-\rho t}(\ln c + \phi \ln S)dt \;\; \text{s.t.} \;\; \dot{S} = MS_t - c_t, \; c_t \geq 0, \; S_t \geq 0, S_0 \; given, \tag{2}$$

where ρ is the social rate of time preference of the planner.

The solution to the problem $\mathcal{P}(1)$ can be written as (e.g., Ayong Le Kama and Schubert, 2004):

$$\frac{\dot{c}^{(1)}}{c^{(1)}} = M - \rho + \phi \frac{c^{(1)}}{S} \tag{3}$$

$$\frac{\dot{S}}{S} = M - \frac{c^{(1)}}{S}. \tag{4}$$

Let us define $z^{(1)} = \frac{c^{(1)}}{S}$ to reduce the system formed by (3) and (4) to the equation:

$$\frac{\dot{z}^{(1)}}{z^{(1)}} = (\phi + 1) z^{(1)} - \rho. \tag{5}$$

The stationary solution of this equation is then given by:

$$\overline{z}^{(1)} = \frac{\rho}{1 + \phi}. \tag{6}$$

The equation is unstable, so the economy will instantaneously switch to its stationary path at the beginning of the time horizon. In this economy, the ratio $\frac{c}{S}$ is constant at the level $\overline{z}$, and c and S growth at the same rate $g^{(1)} = M - \frac{\rho}{1+\phi}$. If the social planner is patient enough so that $(1+\phi) M$ is higher than ρ, i.e. the degree of impatience is inferior to the rate of regeneration plus a term taking into account the weight of the resource stock on the utility, then the economy will be increasing along the optimal path, otherwise g is negative. Then we have $c^{(1)}(t) = c_0 e^{g^{(1)}t} = \left(M - g^{(1)}\right) S_0 e^{g^{(1)}t}$.

3 The worst-case scenario approach

We suppose that there is a possibility that at a random future date T (with marginal density $\omega_t > 0$, i.e. the probability that the discount factor changes at t is positive for any t and $\int_0^\infty \omega_t dt = 1$, i.e. the change of the discount factor happens at a finite date with certainty) the discount factor used to valuate utility will change. It is also assumed that the change in discount factor is a once-for-all phenomenon.

Let $d(t)$ be the discounting factor, and assume that during the first period before T, it is expressed by $e^{-\rho t}$ when the social rate of time-preference applied during the first period before T is equal to $\rho > 0$. Now assume that after T, the social planner chooses the social rate of time-preference in a set of admissible rates $[\delta_{\min}, \rho]$, with $0 < \delta_{\min} < \rho$. We assume that $\delta_{\min} < \rho$ $\forall$ $t > T$. This assumption allows to settle that from the date T the social rate of time-preference applied by the planner is inferior to the one before.

The choice of the social discount rate is known to be an ethical choice. Thus, we think that it is necessary to explicit the normative grounds under which the social discount rate is established. Our purpose is to consider that, at the very long term, taking into account the fundamental uncertainties about the rate of economic growth, the amount of capital that will be accumulated, the degree of diminishing returns, the level and pace of technological progress, the deterioration of the environment, and the effects of natural resource scarcity, it can be difficult to define the appropriate social discount rate. Instead, the attitude of the society can consist in privileging an approach that preserves the well-being of that generations that bear the risk on the future. Therefore, the social planner can consider the more adverse one as relevant scenario for the future, and consequently adopts today the behavior that allows future generations to face more easily the worst-case scenario[4]. That is to say, to apply the lowest possible social discount rate, since it is

[4] See Geoffard (1996) for an interpretation of the worst-case scenario with the introduction of a class of utility functions, called variational utility functions, for which the dynamics of the discount factor obeys a minimum principle.

the one that allows the highest weight to future generations on the welfare function. Then we can sum up the discounting function as:

$$d(t)=\begin{cases} e^{-\rho t} & \text{if } t<T \\ e^{-\delta_{\min}t} & \text{if } t\geq T. \end{cases} \tag{7}$$

3.1 Solution of the model after the change of the discount factor

Consider now the social planner program after the change of the discount factor. The program is the same as the one before T, the differences are on the initial level of the stock of resource and the social discount rate applied in this horizon. The state valuation function of a given remaining stock S_T from time T onwards is therefore given as follow:

$$\mathcal{P}(2) \quad V(S_T)=\max_{c,S}\int_T^{\infty} e^{-\delta_{\min}t}(\ln c+\phi\ln S)dt \;\; s.t. \;\; \dot{S}=MS_t-c_t, \; t>T. \tag{8}$$

The current value Hamiltonian can be written as $\mathcal{H}(c,S,\alpha)=(\ln c+\phi\ln S)+\alpha(MS-c)$.
By analogy, with the solution of the problem $\mathcal{P}(1)$, we obtain for the program $\mathcal{P}(2)$: $\overline{z}^{(2)}=\frac{\delta_{\min}}{1+\phi}$, $g^{(2)}=M-\frac{\delta_{\min}}{1+\phi}$ and $c^{(2)}(T)=\left(M-g^{(2)}\right)S_Te^{-g^{(2)}T}$.
We then can compute the valuation function as:

$$V(S_T)=(1+\phi)\frac{e^{-\delta_{\min}T}}{\delta_{\min}}\ln S_T+\frac{e^{-\delta_{\min}T}}{\delta_{\min}}[\ln\left(M-g^{(2)}\right)+\phi(T+\delta_{\min})+\delta_{\min}g^{(2)}]. \tag{9}$$

We also know that the marginal valuation of the stock is equal to the marginal utility of consumption at time T at which the change in discount rate takes place[5]:

$$\frac{dV(S_T)}{dS_T}=\alpha_T, \tag{10}$$

where α_T is the shadow price of the stock S_T at time T, then we have $\alpha_T=u'\left(c_t^{(2)}\right)=\frac{1}{c_t^{(2)}}$. The condition (10) ensures that it is not necessary to distinguish in S the paths of $\mathcal{P}(2)$ and $\mathcal{P}(3)$[6].

3.2 Solution of the overall model

The overall problem of the social planner program when he takes into account both the uncertainty on the date at which the change of the discount rate occurs and the change in level of the discount rate is defined as (e.g., Dasgupta and Heal, 1974):

$$\mathcal{P}(3) \quad \begin{cases} \max\int_0^{\infty}\omega_T\left\{\int_0^T(\ln c+\phi\ln S)e^{-\rho t}dt+V(S_T)e^{-\rho T}\right\}dT \\ \quad s.t. \; \dot{S}=MS-c, c_t\geq 0, S_t\geq 0, \omega_T>0 \; \forall t. \end{cases} \tag{11}$$

[5] For the proof, see Beltratti, Chichilnisky and Heal (1998) proposition 6.
[6] Therefore, we can deduce $c_t^{(2)}$ with respect to S_t as: $c_t^{(2)}=\left(M-g^{(2)}\right)S_t$, since there can not exist a jump in the stock.

Write $\Omega_t = \displaystyle\int_0^t \omega_\tau d\tau$ and integrate by parts the maximand in (11) to reformulate the problem $\mathcal{P}(3)$ as:

$$\begin{aligned} & \max \int_0^\infty e^{-\rho t}[(\ln c + \phi \ln S)\,\Omega_t + \omega_t V(S_t)]dt \\ s.t.\ \dot{S} &= MS - c, c_t \geq 0, S_t \geq 0, \text{ and } S_0 \text{ given.} \end{aligned}$$

The current value Hamiltonian is then $\mathcal{H}(c, S, \gamma, \omega, \Omega) = \Omega(\ln c + \phi \ln S) + \omega V(S) + \gamma(MS - c)$, and the first order conditions are:

$$\Omega \frac{1}{c^{(3)}} = \gamma, \tag{12}$$

$$\frac{\dot{\gamma}}{\gamma} = \rho - M - \phi\frac{c^{(3)}}{S} - \frac{\omega c V'}{\Omega}. \tag{13}$$

Differentiating the first optimality condition, using equation (13) and (10), and assuming that the marginal density ω_t of the random future date t at which the change in discount rate occurs is a Poisson process with constant parameter λ, $\frac{\omega_t}{\Omega_t} = \lambda\ \forall t$, allow to display the differential equations characterizing the evolution of the economy:

$$\frac{\dot{c}^{(3)}}{c^{(3)}} = M - \rho + \phi\frac{c^{(3)}}{S} + \lambda\left[1 - \frac{c^{(3)}}{\left(M - g^{(2)}\right)S}\right] \tag{14}$$

$$\frac{\dot{S}}{S} = M - \frac{c^{(3)}}{S}. \tag{15}$$

Defining $z^{(3)} = \frac{c^{(3)}}{S}$, the dynamic system formed by (14) and (15) reduces to an unique equation in $z^{(3)}$ given by:

$$\frac{\dot{z}^{(3)}}{z^{(3)}} = \lambda - \rho + \left[(\phi + 1) - \frac{\lambda}{M - g^{(2)}}\right] z^{(3)}. \tag{16}$$

Assuming that $\delta_{\min} > \lambda$, the stationary solution of this equation is then given by $\overline{z}^{(3)} = \frac{\delta_{\min}}{1+\phi}\frac{\rho-\lambda}{\delta_{\min}-\lambda}$ and we obtain $g^{(3)} = M - \frac{\delta_{\min}}{1+\phi}\frac{\rho-\lambda}{\delta_{\min}-\lambda}$ and $c_T^{(3)} = \left(M - g^{(3)}\right)S_T$.

3.3 Comparison of optimal paths

We can compare the paths of the determinist economy, $\mathcal{P}(1)$, and the overall model, $\mathcal{P}(3)$, since they obey to the same resource constraint. The comparison of the growth rate and initial levels of consumption allows to determine which economy follows a path that sacrificed less generations. We will take the trajectory of the determinist economy as the reference path. Moreover, we consider that the social planner is more conservative in terms of resource if the ratio of the consumption on the resource stock is lower along the economic path.

Since we assume that $\delta_{\min} > \lambda$, it is straightforward to see that $g^{(3)} = M - \frac{\delta_{\min}}{1+\phi}\frac{\rho-\lambda}{\delta_{\min}-\lambda} < g^{(2)} = M - \frac{\delta_{\min}}{1+\phi}$ since $\rho > \delta_{\min}$. Moreover, one can see straightforwardly that $g^{(1)} < g^{(2)}$.

For the comparison of $g^{(3)}$ and $g^{(1)}$, we must consider two cases. In case one, if $\delta_{\min}\frac{\rho-\lambda}{\delta_{\min}-\lambda} > \rho$, then $g^{(3)} < g^{(1)}$ and $c_0^{(3)} > c_0^{(1)}$. Thus the optimal consumption path of $\mathcal{P}(3)$ will remain above the one of $\mathcal{P}(1)$ all the long of trajectories before the change occurs at T. At the date of the change in discounting, the consumption level in the overall model takes down to a lower level, $c_T^{(2)}$, and hereafter grows at a higher rate $g^{(2)}$. Under this case, the generations coming just after T are sacrificed in the program taking into account the change in the discount rate. This evolution means that after being less cautious in the beginning, when the awareness about resource constraint becomes high, decisions are taken to make sacrifice at this time and to preserve the future. In that case, the lower is $\delta_{\min}$, the stronger is the growth rate which result in a smaller number of generations that is sacrificed with respect to the determinist economy. Figure 1 illustrates consumption paths in this case.

In case two, if $\delta_{\min}\frac{\rho-\lambda}{\delta_{\min}-\lambda} < \rho$, then $g^{(3)} > g^{(1)}$ and $c_0^{(3)} < c_0^{(1)}$. In that case, between the date 0 and T, the economy with uncertainty will start with a lower level of consumption and will follow a more conservative path along this period. At the date T, the consumption level takes down also to a lower level and hereafter follows a path with a higher growth rate. The overall economy with change in the discount factor is therefore more cautious in the beginning until the change arrives. In that case, it is the generations before the change occurs at date T that are sacrificed. That is to say, the worst-case scenario allows in that case to follow an economic path that pay more attention to future generations interests. Figure 2 illustrates this case.

The interesting question becomes therefore, not to know the level of discount rate to apply, but to know when it is optimal to make the change in discount factor. Our approach is suggesting that it is better in terms of intergenerational equity to behave as if the future is beginning today and to make the change immediately. As it is shown in the program $\mathcal{P}(2)$, taking a lower discount rate allows to follow an optimal path with a higher growth rate, but it implies also to make the sacrifice in terms of consumption level today if we make the change today in order to preserve possibilities of consumption paths for future generations.

4 Concluding remarks

This paper develops a prescriptive approach on the debate on discounting the very long term. Our purpose was to tackle the issue of how to discount the distant-future when we consider environmental questions and the uncertainty inherent to this horizon.

The adoption of the worst-case scenario approach leads to choose the lowest possible discount rate. This gives more value to future generations well-being on the social welfare function but also offers

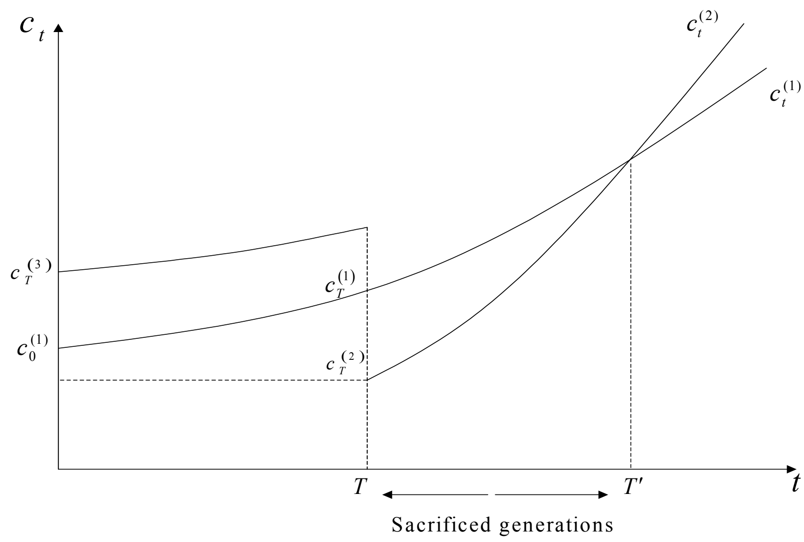

Figure 1: Consumption paths: case 1: $\delta_{\min}\frac{\rho-\lambda}{\delta_{\min}-\lambda} > \rho$

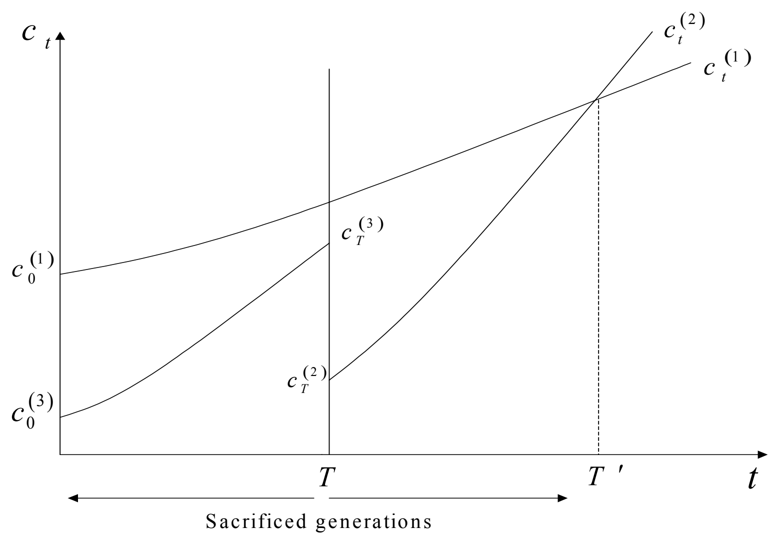

Figure 2: Consumption paths: case 2: $\delta_{\min}\frac{\rho-\lambda}{\delta_{\min}-\lambda} < \rho$

room for manoeuvre if events turn bad. Finally, considering the uncertainty on the date at which the change on the discount rate should be made, our model invites to make the change immediately.

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